

DUALIZING COMPLEXES AND HOMOMORPHISMS VANISHING IN KOSZUL HOMOLOGY

JAVIER MAJADAS

ABSTRACT. Let C be a semidualizing complex over a noetherian local ring A . If there exists a local homomorphism with source A satisfying some homological properties, then C is dualizing.

1. INTRODUCTION

There is a number of characterizations of properties (of homological type) of noetherian local rings of positive characteristic in terms (of homological properties) of the Frobenius homomorphism. We start with [20]:

Theorem (Kunz) *Let A be a noetherian local ring containing a field of characteristic $p > 0$, and let $\phi : A \rightarrow A, \phi(a) = a^p$ be the Frobenius homomorphism. We denote by ${}^\phi A$ the ring A considered as A -module via ϕ . The following conditions are equivalent:*

- (i) A is regular
- (ii) ${}^\phi A$ is a flat A -module.

Some years later, Kunz theorem was improved by Rodicio [25] as follows: if the flat dimension $fd_A({}^\phi A) < \infty$, then A is regular.

So we can think if similar characterizations for the properties complete intersection, Gorenstein and Cohen-Macaulay exist. For complete intersections the result was obtained in [12], characterizing complete intersections rings by the finiteness of the complete intersection dimension [8] of its Frobenius homomorphism, and a similar characterization was also found for the Cohen-Macaulay property in [26].

We will examine now in more detail the case of the Gorenstein property. A first result was obtained by Herzog [17] (see also [15, Theorem 1.1] and [26, Proposition 6.1]):

Theorem (Herzog) *Let A be a noetherian local ring containing a field of characteristic $p > 0$, and let ϕ be its Frobenius homomorphism. Assume that ϕ is finite. The following conditions are equivalent:*

- (i) A is Gorenstein
- (ii) $Ext_A^i({}^{\phi^r} A, A) = 0$ for all $i > 0$ and infinitely many $r > 0$.

This result was improved in [18], removing in particular the annoying finiteness hypothesis on ϕ :

Theorem (Iyengar, Sather-Wagstaff) *Let A be a noetherian local ring containing a field of characteristic $p > 0$, and let ϕ be its Frobenius homomorphism.*

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The following conditions are equivalent:

- (i) A is Gorenstein
- (ii) $G\text{-dim}_A(\phi^r A) < \infty$ for some integer $r > 0$.

Here $G\text{-dim}$ denotes the Gorenstein dimension introduced by Auslander and Bridger in [2] (properly speaking, a modification of the original definition using Cohen factorizations [6, p.254], [18, Definition 3.3]).

Over the last years, some research was conducted in order to extend these results from the particular case of the Frobenius homomorphism to larger classes of homomorphisms. A first step was to consider contracting endomorphisms. An endomorphism f of a noetherian local ring (A, \mathfrak{m}, k) is contracting if for any integer $s > 0$ there exist an integer $r > 0$ such that $f^r(\mathfrak{m}) \subset \mathfrak{m}^s$. The Frobenius homomorphism is an example of contracting endomorphism. If f is a contracting endomorphism on a noetherian local ring A , then A must contain a field (of fixed elements), but unlike the case of the Frobenius homomorphism, it can be of characteristic zero. The above results for regularity were extended to contracting endomorphisms in [19, Proposition 2.6]. For the complete intersection property they were extended (even in an improved form) in [11], [9].

The Gorenstein case was studied first in [18]. In fact, they obtain the theorem stated above as a consequence of the more general:

Theorem (Iyengar, Sather-Wagstaff) *Let A be a noetherian local ring and $\phi : A \rightarrow A$ a contracting endomorphism. Then the following conditions are equivalent:*

- (i) A is Gorenstein
- (ii) $G\text{-dim}_A(\phi^r A) < \infty$ for some integer $r > 0$.

Subsequently, in [24] this result was extended to the more general context of G -dimension over a semidualizing complex C as defined in [13]. It is obtained in particular:

Theorem (Nasseh, Sather-Wagstaff) *Let A be a noetherian local ring, C a semidualizing complex over A and $\phi : A \rightarrow A$ a contracting endomorphism. The following conditions are equivalent:*

- (i) C is a dualizing complex.
- (ii) $G_C\text{-dim}(\phi^r A) < \infty$ for infinitely many $r > 0$.

This result generalizes the “classical” case: the Gorenstein dimension of [2] is the particular case of $G_C\text{-dim}$ obtained by taking $C = A$ (which is always a semidualizing complex), and a ring A is Gorenstein if and only if the semidualizing complex A is dualizing.

A second step in the extension of these results to larger classes of homomorphisms was initiated in [21]. The purpose in that paper was not so much to extend the known results to larger classes of homomorphisms as to understand what a homomorphism must verify in order to be a “test homomorphism” for these properties of local rings. In that paper a new class of homomorphisms, the ones with the h_2 -vanishing property, was introduced. A local homomorphism $f : (A, \mathfrak{m}, k) \rightarrow (B, \mathfrak{n}, l)$ of noetherian local rings is said to have the h_2 -vanishing property if the canonical homomorphism between the first Koszul homology modules associated to minimal sets of generators of their maximal ideals

$$H_1(\mathfrak{m}) \otimes_k l \rightarrow H_1(\mathfrak{n})$$

is zero. Any contracting endomorphism has a power which has the h_2 -vanishing property, but h_2 -vanishing homomorphisms are not necessarily endomorphisms, and they can be defined on rings that do not contain a field. Moreover, unlike the class of contracting endomorphisms, the class of h_2 -vanishing homomorphisms contains at once the two main test homomorphisms: the Frobenius endomorphism and the canonical epimorphism of a local ring into its residue field.

In order to see, even in the case of an endomorphism, the difference between h_2 -vanishing and contracting, consider a complete local ring (A, \mathfrak{m}, k) and a contracting endomorphism ϕ of A . We assume for simplicity that $\phi(\mathfrak{m}) = \phi^1(\mathfrak{m}) \subset \mathfrak{m}^2$. Take a regular local ring (R, \mathfrak{n}, k) of minimal dimension such that $A = R/I$ (i.e., $\dim R = \text{emb.dim } A$), and a contracting endomorphism φ of R making commutative the diagram [23, 3.2.1, 3.2.4]

$$\begin{array}{ccc} R & \xrightarrow{\varphi} & R \\ \downarrow & & \downarrow \\ A & \xrightarrow{\phi} & A \end{array}$$

(details can be seen in [21, Example 3.ii]).

Then the homomorphism induced by ϕ

$$H_1(\mathfrak{m}) \otimes_k {}^\phi k \rightarrow H_1(\mathfrak{m})$$

can be identified with the canonical homomorphism induced by φ

$$I/\mathfrak{n}I \otimes_k {}^\phi k \rightarrow I/\mathfrak{n}I.$$

Since φ is contracting, by the Artin-Rees lemma some power of it verifies $\varphi^r(I) \subset \mathfrak{n}I$, and so ϕ^r has the h_2 -vanishing property. But the contracting property is not only a condition on the images of I , but on the images of \mathfrak{n} . For instance, any local homomorphism which factorizes through a regular local ring has the h_2 -vanishing property.

Our purpose in this paper is to extend the above result of Nasseh and Sather-Wagstaff to h_2 -vanishing homomorphisms. In order to achieve it, instead of working with G_C -dim, we consider a different definition, G_C^* -dim (see Definition 1). Both definitions are related in the same way that Gorenstein dimension G -dim is related to upper Gorenstein dimension G^* -dim [27], [4, §8]. They share the usual properties (see Propositions 3 and 3*), but we do not know if the finiteness of G_C -dim is equivalent to the finiteness of G_C^* -dim.

We obtain:

Theorem 6 *Let $\varphi : A \rightarrow B$ be a local homomorphism and C a semidualizing complex over A . Assume that φ has the h_2 -vanishing property. The following conditions are equivalent:*

- (i) *C is a dualizing complex.*
- (ii) *$G_C^*\text{-dim}(B) < \infty$.*

A note on terminology. Since we are interested only in the finiteness of G_C -dim and not in its precise value, we use the terminology of derived C -reflexivity instead of finite G_C -dim.

2. NOTATION FOR COMPLEXES

All rings in this paper will be noetherian and local.

We will follow the conventions for complexes generally used in this context (see e.g. [13]). For convenience of the reader we will briefly recall some notation. Let A be a ring. A complex of A -modules will be a sequence of A -module homomorphisms

$$X = \dots \rightarrow X_{n+1} \xrightarrow{d_{n+1}} X_n \xrightarrow{d_n} X_{n-1} \rightarrow \dots$$

such that $d_n d_{n+1} = 0$ for all n . If m is an integer, $\Sigma^m X$ will be the complex with $(\Sigma^m X)_n = X_{n-m}$, $d_n^{\Sigma^m X} = (-1)^m d_{n-m}^X$ for all n .

The derived category of the category of A -modules will be denoted by $\mathbf{D}(A)$. For $X, Y \in \mathbf{D}(A)$, we will write $X \simeq Y$ if X and Y are isomorphic in $\mathbf{D}(A)$, and $X \sim Y$ if $X \simeq \Sigma^m Y$ for some integer m . Sometimes we will consider an A -module as a complex concentrated in degree 0. The full subcategory of $\mathbf{D}(A)$ consisting of complexes homologically finite, that is, complexes X such that $H(X)$ is an A -module of finite type, will be denoted by $\mathbf{D}_b^f(A)$.

The left derived functor of the tensor product of complexes of A -modules will be denoted by $- \otimes_A^{\mathbf{L}} -$, and similarly $\mathbf{RHom}_A(-, -)$ will denote the right derived functor of the Hom functor on complexes of A -modules. We say that a complex $X \in \mathbf{D}(A)$ is of finite projective (respectively, injective) dimension if there exists a bounded complex Y (that is, $Y_n = 0$ for $|n| \gg 0$) of projective (respectively, injective) modules such that $X \simeq Y$. We will denote it by $\mathrm{pd}_A(X) < \infty$ (respectively, $\mathrm{id}_A(X) < \infty$).

3. DERIVED REFLEXIVITY

Let $X, C \in \mathbf{D}_b^f(A)$. We say that X is *derived C -reflexive* if $\mathbf{RHom}_A(X, C) \in \mathbf{D}_b^f(A)$ and the canonical biduality morphism

$$X \rightarrow \mathbf{RHom}_A(\mathbf{RHom}_A(X, C), C)$$

is an isomorphism in $\mathbf{D}_b^f(A)$ [13, 2.7], [10, §2].

We will say that $C \in \mathbf{D}_b^f(A)$ is a *semidualizing complex* [13, Definition 2.1] if A is derived C -reflexive, that is, if the homothety morphism

$$A \rightarrow \mathbf{RHom}_A(C, C)$$

is an isomorphism in $\mathbf{D}_b^f(A)$. If C is a semidualizing complex and $X \in \mathbf{D}_b^f(A)$, then X is derived C -reflexive if and only if $X \simeq \mathbf{RHom}_A(\mathbf{RHom}_A(X, C), C)$ [10, Theorem 3.3]. We will give precise references of all the results we need on derived reflexivity, but the reader may consult [13], [14] for a systematic study.

A *dualizing complex* is a semidualizing complex of finite injective dimension.

We now introduce a modification of derived C -reflexivity. We call it derived C -reflexivity*, since it is related to derived C -reflexivity in the same way that upper Gorenstein dimension $\mathrm{G}^*\text{-dim}$ is related to Gorenstein dimension $\mathrm{G}\text{-dim}$ [27], [4, §8].

A local homomorphism $(A, \mathfrak{m}, k) \rightarrow (R, \mathfrak{p}, l)$ is *weakly regular* if it is flat and the closed fiber $R \otimes_A k$ is a regular local ring. Let $f : (A, \mathfrak{m}, k) \rightarrow (B, \mathfrak{n}, l)$ be a local homomorphism. A *regular factorization* of f is a factorization $A \xrightarrow{i} R \xrightarrow{p} B$ of f

where i is weakly regular and p is surjective. If B is complete, then f has a regular factorization with R complete [7].

Definition 1. If C is a semidualizing complex over a ring A , a C -deformation of A will be a pair (Q, E) consisting in a surjective homomorphism of (local) rings $Q \rightarrow A$ and a semidualizing complex $E \in \mathbf{D}_b^f(Q)$ such that $\mathbf{R}\mathrm{Hom}_Q(A, E) \sim C$. In this case, by [13, Theorem 6.1 and Observation 2.4], the Q -module A is derived E -reflexive.

Let C be a semidualizing complex over A , and $X \in \mathbf{D}_b^f(A)$. We will say that X is *derived C -reflexive** if there exists a weakly regular homomorphism $A \rightarrow A'$ and a $C \otimes_A A'$ -deformation (Q, E) of A' (note that $C \otimes_A A' = C \otimes_A^{\mathbf{L}} A'$ is a semidualizing complex over A' [13, Theorem 5.6]) such that $\mathrm{pd}_Q(X \otimes_A A') < \infty$.

Let $\varphi : A \rightarrow B$ be a local homomorphism, C be a semidualizing complex over A , $X \in \mathbf{D}_b^f(B)$. We will say that X is *derived C - φ -reflexive** if there exists a regular factorization $A \rightarrow R \rightarrow \hat{B}$ such that the complex of R -modules $X \otimes_B \hat{B}$ is derived $C \otimes_A R$ -reflexive*, where \hat{B} is the completion of B .

Proposition 2. *Let C be a semidualizing complex over A , and $X \in \mathbf{D}_b^f(A)$. If X is derived C -reflexive*, then it is derived C -reflexive.*

Proof. Let $A \rightarrow A'$ be a weakly regular homomorphism, (Q, E) a $C' := C \otimes_A A'$ -deformation of A' such that $\mathrm{pd}_Q(X \otimes_A A') < \infty$. By [13, Proposition 2.9], $X \otimes_A A'$ is derived E -reflexive and then, by [13, Theorem 6.5], $X \otimes_A A'$ is derived C' -reflexive. Then faithfully flat base change [13, Theorem 5.10] gives that X is derived C -reflexive. \square

We do not know if the reciprocal of Proposition 2 is true, even in the (classical) case $C = A$. However the usual characterization of dualizing complexes in terms of derived reflexivity of the residue field also remain valid for derived reflexivity* (in the case $C = A$ this is the theorem by Auslander and Bridger saying that a ring A is Gorenstein if and only if the Gorenstein dimension of any module of finite type is finite if and only if the Gorenstein dimension of its residue field is finite [2, Theorem 4.20 and its proof]; see also [18, Theorem 6.1]):

Proposition 3. [13, Proposition 8.4, Remark 8.5] *Let C be a semidualizing complex over A . The following are equivalent:*

- (i) C is dualizing.
- (ii) Any $X \in \mathbf{D}_b^f(A)$ is derived C -reflexive.
- (iii) The residue field k of A is derived C -reflexive.

Proposition 3* Let C be a semidualizing complex over A . The following are equivalent:

- (i) C is dualizing.
- (ii*) Any $X \in \mathbf{D}_b^f(A)$ is derived C -reflexive*.
- (iii*) The residue field k of A is derived C -reflexive*.

Proof. By Propositions 2 and 3, we only have to show (i) \Rightarrow (ii*). Let \hat{A} be the completion of A and $Q \rightarrow \hat{A}$ a surjection where Q is a regular local ring. Let D be a dualizing complex over Q ($D \sim Q$). Then $\mathbf{R}\mathrm{Hom}_Q(\hat{A}, D)$ is a dualizing complex over \hat{A} ([16, V.2.4] or [13, Corollary 6.2]). Also, $C \otimes_A \hat{A}$ is a dualizing complex over \hat{A} ([16, V.3.5]), so $\mathbf{R}\mathrm{Hom}_Q(\hat{A}, D) \sim C \otimes_A \hat{A}$ by [16, V.3.1].

Therefore (Q, D) is a $C \otimes_A \hat{A}$ -deformation of \hat{A} . Since Q is regular, for any $X \in \mathbf{D}_b^f(A)$ we have $\mathrm{pd}_Q(X \otimes_A \hat{A}) < \infty$, and so X is derived C -reflexive*. \square

This result still holds for derived C - φ -reflexivity*:

Proposition 4. *Let C be a semidualizing complex over A . The following are equivalent:*

- (i) C is dualizing.
- (ii) For any local homomorphism $\varphi : A \rightarrow B$, any $X \in \mathbf{D}_b^f(B)$ is derived C - φ -reflexive*.
- (iii) There exists a local homomorphism $\varphi : A \rightarrow B$, such that the residue field l of B is derived C - φ -reflexive*.

Proof. (i) \Rightarrow (ii) Let $\varphi : A \rightarrow B$ be a local homomorphism and let $A \rightarrow R \rightarrow \hat{B}$ be a regular factorization with R complete. Since C is a dualizing complex over A and i is flat with Gorenstein (in fact regular) closed fiber, then $C \otimes_A R$ is a dualizing complex over R [5, Theorem 5.1, Proposition 4.2]. Therefore the result follows from Proposition 3*.

(iii) \Rightarrow (i) Let $A \xrightarrow{i} R \xrightarrow{p} \hat{B}$ be a regular factorization, $R \rightarrow R'$ a weakly regular homomorphism, and (Q, E) a $C \otimes_A R'$ -deformation of R' such that $\mathrm{pd}_Q(l \otimes_R R') < \infty$. Since $R \rightarrow R'$ is weakly regular, its closed fiber $l \otimes_R R'$ is regular. Then Q is a regular local ring (it follows e.g. from the change of rings spectral sequence

$$E_p^2 q = \mathrm{Tor}_p^{l \otimes_R R'}(\mathrm{Tor}_q^Q(l \otimes_R R', l'), l') \Rightarrow \mathrm{Tor}_q^Q(l', l')$$

where l' is the residue field of Q and $l \otimes_R R'$).

We deduce that $\mathrm{id}_Q(E) < \infty$, and so the semidualizing complex E is dualizing. Then $C \otimes_A R' \sim \mathbf{R}\mathrm{Hom}_Q(R', E)$ is also dualizing [16, V.2.4]. Since $A \rightarrow R'$ is flat, it is easy to see that C is dualizing (or use the stronger result [5, Theorem 5.1]). \square

Definition 5. [21, Definition 1] Let $f : (A, \mathfrak{m}, k) \rightarrow (B, \mathfrak{n}, l)$ be a local homomorphism. Let $H_*(\mathfrak{m})$ (respectively, $H_*(\mathfrak{n})$) be the Koszul homology associated to a minimal system of generators of the ideal \mathfrak{m} of A (respectively, the ideal \mathfrak{n} of B). We say that f has the h_2 -vanishing property if the canonical homomorphism induced by f

$$H_1(\mathfrak{m}) \otimes_k l \rightarrow H_1(\mathfrak{n})$$

vanishes.

By [1, 15.12] (see [23, 2.5.1]), this homomorphism between Koszul homology modules can be written in terms of André-Quillen homology [1] as the canonical homomorphism

$$H_2(A, k, l) \rightarrow H_2(B, l, l).$$

As we saw in the Introduction, a suitable power of any contracting endomorphism has the h_2 -vanishing property (in fact, if $f : (A, \mathfrak{m}, k) \rightarrow (A, \mathfrak{m}, k)$ is a contracting endomorphism, for any integer n there exists an integer s such that f^s has the h_n -vanishing property, in the sense that the morphism of functors $H_n(A, k, -) \rightarrow H_n(A, k, -)$ vanishes [22, Proposition 10]).

Theorem 6. *Let $\varphi : A \rightarrow B$ be a local homomorphism and C a semidualizing complex over A . Assume that φ has the h_2 -vanishing property. If (and only if) B is derived C - φ -reflexive*, then C is dualizing.*

Proof. The “only if” part is a consequence of Proposition 4.

Assume then that B is derived C - φ -reflexive*. Consider a diagram of local homomorphisms

$$\begin{array}{ccccc}
 & & Q & & \\
 & & \searrow & & \\
 & R & \xrightarrow{\rho} & R' & \\
 \alpha \nearrow & & \searrow \pi & & \searrow \pi' \\
 A & \xrightarrow{\varphi} & B & \xrightarrow{\beta} & \hat{B} \longrightarrow \hat{B} \otimes_R R'
 \end{array}$$

where α and ρ are weakly regular, π is surjective and (Q, E) is a $C \otimes_A R'$ -deformation of R' such that $\text{pd}_Q(\hat{B} \otimes_R R') < \infty$. We will see first that Q is a regular local ring repeating an argument in the proof of [21, Proposition 6].

Let l be the residue field of Q and $\hat{B} \otimes_R R'$. The commutative square

$$\begin{array}{ccc}
 A & \xrightarrow{\alpha} & R \\
 \downarrow \varphi & & \downarrow \pi \\
 B & \xrightarrow{\beta} & \hat{B}
 \end{array}$$

induces a commutative square

$$\begin{array}{ccc}
 H_2(A, l, l) & \xrightarrow{\tilde{\alpha}} & H_2(R, l, l) \\
 \downarrow \tilde{\varphi} & & \downarrow \tilde{\pi} \\
 H_2(B, l, l) & \xrightarrow{\tilde{\beta}} & H_2(\hat{B}, l, l).
 \end{array}$$

We have $\tilde{\varphi} = 0$ since φ has the h_2 -vanishing property (we have used that if $k \rightarrow l$ is a field extension we have $H_n(k, l, l) = 0$ for all $n \geq 2$ [1, 7.4]; so if $A \rightarrow k \rightarrow l$ are ring homomorphisms with k and l fields, from the Jacobi-Zariski exact sequence [1, 5.1] we obtain $H_n(A, k, l) = H_n(A, l, l)$ for all $n \geq 2$; finally, $H_n(A, k, k) \otimes_k l = H_n(A, k, l)$ for all n by [1, 3.20]).

Since α is weakly regular, by [21, Lemma 5], $\tilde{\alpha}$ is an isomorphism, and so $\tilde{\pi} = 0$. Consider now the commutative square

$$\begin{array}{ccc}
 H_2(R, l, l) & \xrightarrow{\tilde{\rho}} & H_2(R', l, l) \\
 \downarrow 0 & & \downarrow \tilde{\pi}' \\
 H_2(\hat{B}, l, l) & \longrightarrow & H_2(\hat{B} \otimes_R R', l, l).
 \end{array}$$

Again by [21, Lemma 5], $\tilde{\rho}$ is an isomorphism, and then $\tilde{\pi}' = 0$. So the composition

$$H_2(Q, l, l) \rightarrow H_2(R', l, l) \xrightarrow{\tilde{\pi}'} H_2(\hat{B} \otimes_R R', l, l)$$

vanishes. But by [3], $\mathrm{pd}_Q(\hat{B} \otimes_R R') < \infty$ implies that

$$H_2(Q, l, l) \rightarrow H_2(\hat{B} \otimes_R R', l, l)$$

is injective. Therefore $H_2(Q, l, l) = 0$, and then Q is regular by [1, 6.26].

Now the proof finishes as the proof of Proposition 4: since $\mathrm{id}_Q(E) < \infty$, the semidualizing complex E is dualizing; then $C \otimes_A R' \sim \mathbf{R}\mathrm{Hom}_Q(R', E)$ is also dualizing [16, V.2.4] and since $\rho\alpha$ is flat, we deduce that C is dualizing. \square

Remark 7. If a homomorphism in a composition has h_2 -vanishing property, then so has the composition. Therefore Theorem 6 can also be stated as follows:

Let $\varphi : A \rightarrow B$ be a local homomorphism and C a semidualizing complex over A . Assume that φ has the h_2 -vanishing property. If there exists a local homomorphism $\phi : B \rightarrow S$ such that S is derived C - $\phi\varphi$ -reflexive*, then C is dualizing.

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DEPARTAMENTO DE ÁLGEBRA, FACULTAD DE MATEMÁTICAS, UNIVERSIDAD DE SANTIAGO DE COMPOSTELA, E15782 SANTIAGO DE COMPOSTELA, SPAIN
E-mail address: j.majadas@usc.es